

Network analysis begins with data that describes the set of relationships among the members of a system. The goal of analysis is to obtain from the low-level relational data a higher-level description of the structure of the system which identifies various kinds of patterns in the set of relationships. These patterns will be based on the way individuals are related to other individuals in the network. Some approaches to network analysis look for clusters of individuals who are tightly connected to one another; some look for sets of individuals who have similar patterns of relations to the rest of the network. Our work has emphasized the use of spectral methods for finding structure in social and other networks, a focus on graph theoretic rather than statistical methods, the development of a unified, coherent theoretical foundation which shows the connections to other fields, and the insistence on concepts and procedures that scale well to very large networks. Spectral methods construct a continuous multidimensional representation of the network in which the coordinates of the individuals can be further analysed to obtain a variety of kinds of information about them and their relation to the rest of the network.

Spectral methods (eigendecomposition) have been a part of graph theory for over a century. Network researchers have used spectral methods either implicitly or explicitly since the late 1960's, when computers became generally accessible in most universities. The eigenvalues of a network are intimately connected to important topological features such as maximum distance across the network (diameter), presence of cohesive clusters, long paths and bottlenecks, and how random the network is. The associated eigenvectors can be used as a natural coordinate system for graph visualization; they also provide methods for discovering clusters and other local features. Eigenvector coordinates may be analysed in combination with other node attributes either given as part of the data (e.g. sex) or derived from the network (e.g. degree) to find relationships in large, complex multivariate networks

In particular, the Normal spectrum has become the centre of our research and developmental efforts since its natural handling of bipartite graphs has allowed us to examine multi-modal networks, which we have recently used to address medical problems such as relationships among symptoms and exposures, and co-occurrences of types of cancers in families. The use of spectral methods allows *analytic visualization*. To extend this approach, we have developed methods that build upon spectral results, making it possible, for example, to bring node and link characteristics into the analysis. These are demonstrated in figure 1, which shows Normal spectral results for people, symptoms and exposures in which the locations of people on one eigenvector have been quantized into deciles. Figure 2a shows the relation between the decile people are in and the symptoms they report. Figure 2b shows the same deciles of people and the exposures they report. This clustering of types of symptoms and associated clustering of types of exposures is a new result.

Figure 3 shows two dimensions from the Normal spectrum of a 3-mode dataset describing 1440 families, their members, and co-occurrences of several types of cancer. Deciles based on this analysis reveal different patterns of co-occurrences in the bottom (Q1) and top (Q10) deciles of families – something not previously observed.

LINK: "GP scexpe" Normal  
Evec 1 Eval -1.0  
Evec 5 Eval 0.345  
Evec 3 Eval 0.492  
# Nodes = 1493

Deciles of People+Sym+Exp

- 1 1Q
- 2 2Q
- 3 3Q
- 4 4Q
- 5 5Q
- 6 6Q
- 7 7Q
- 8 8Q
- 9 9Q
- 10 10Q

11 Symptoms  
12 Exposures

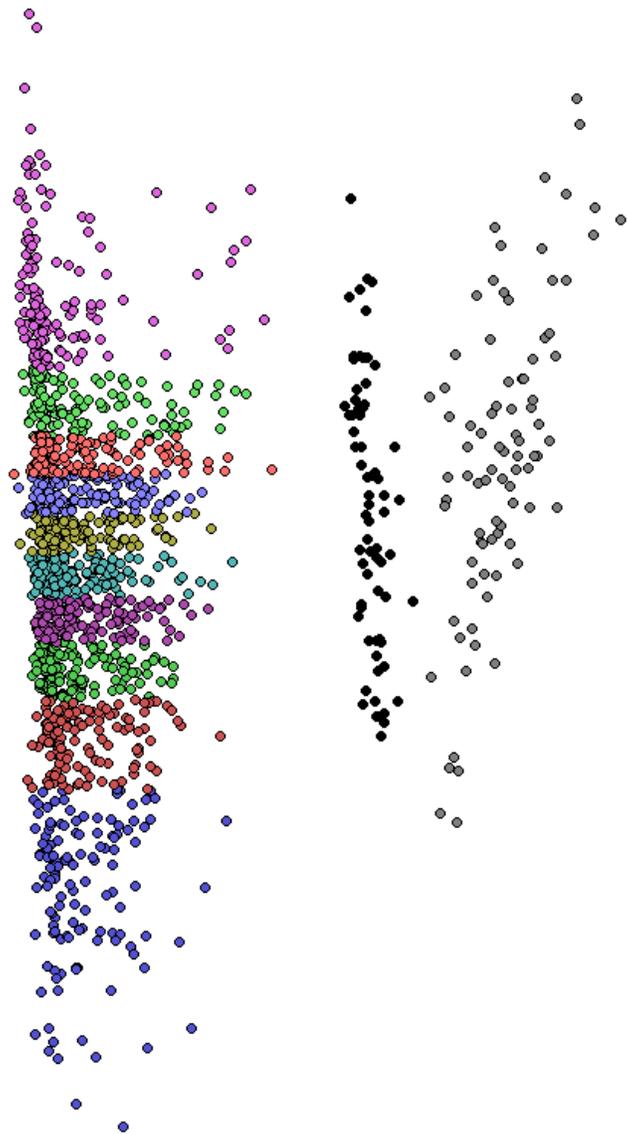
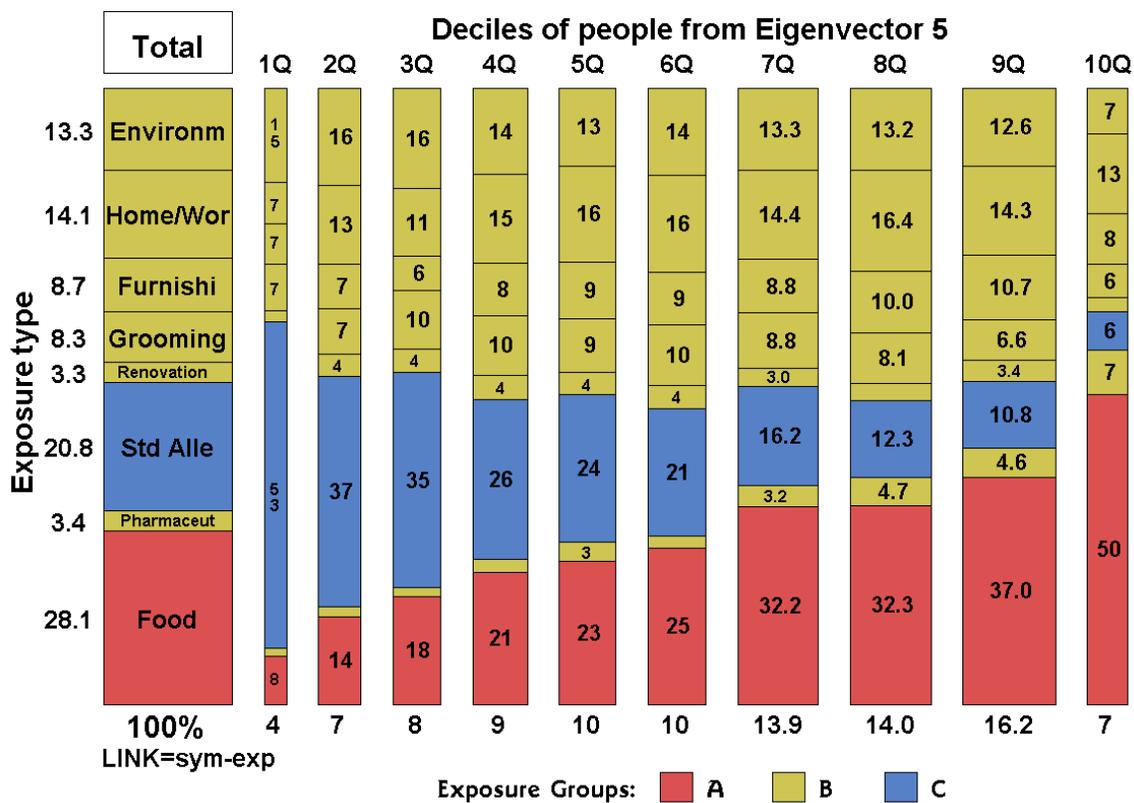
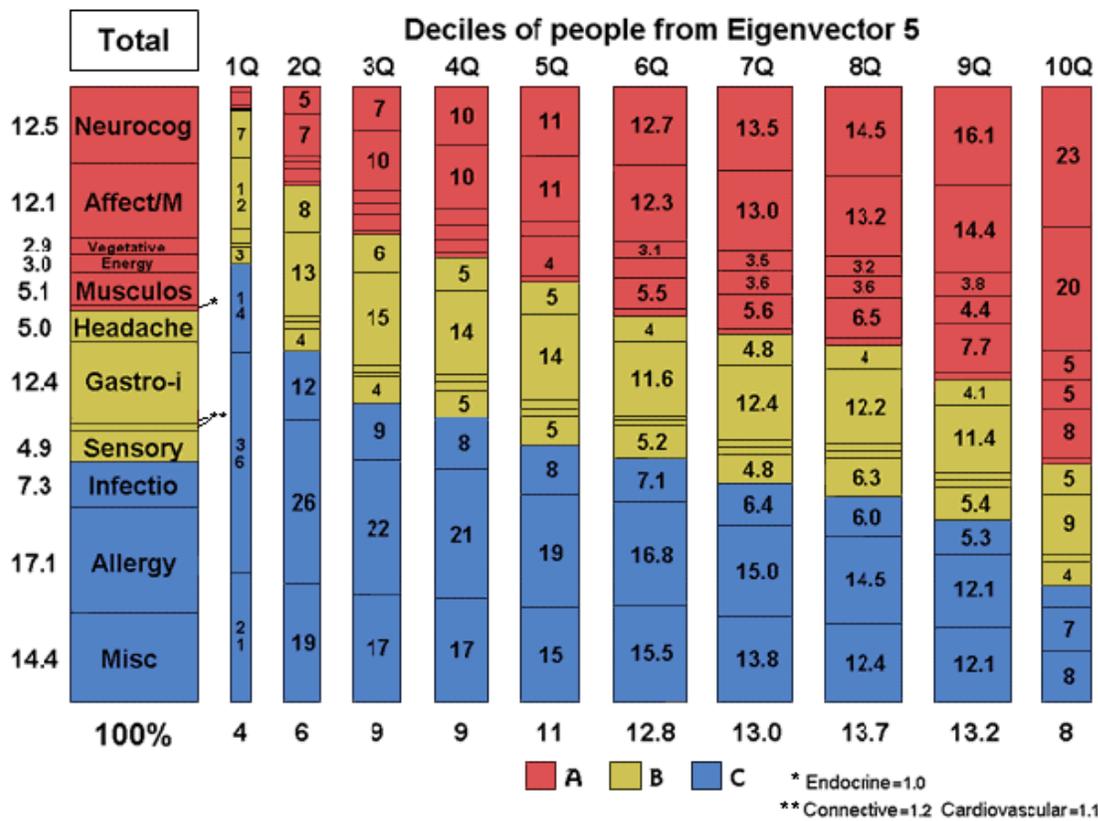


Figure 1



Figures 2a,b

**LINK: "Cancers of FDRs" Normal**

**Evec 1 Eval -1.0**

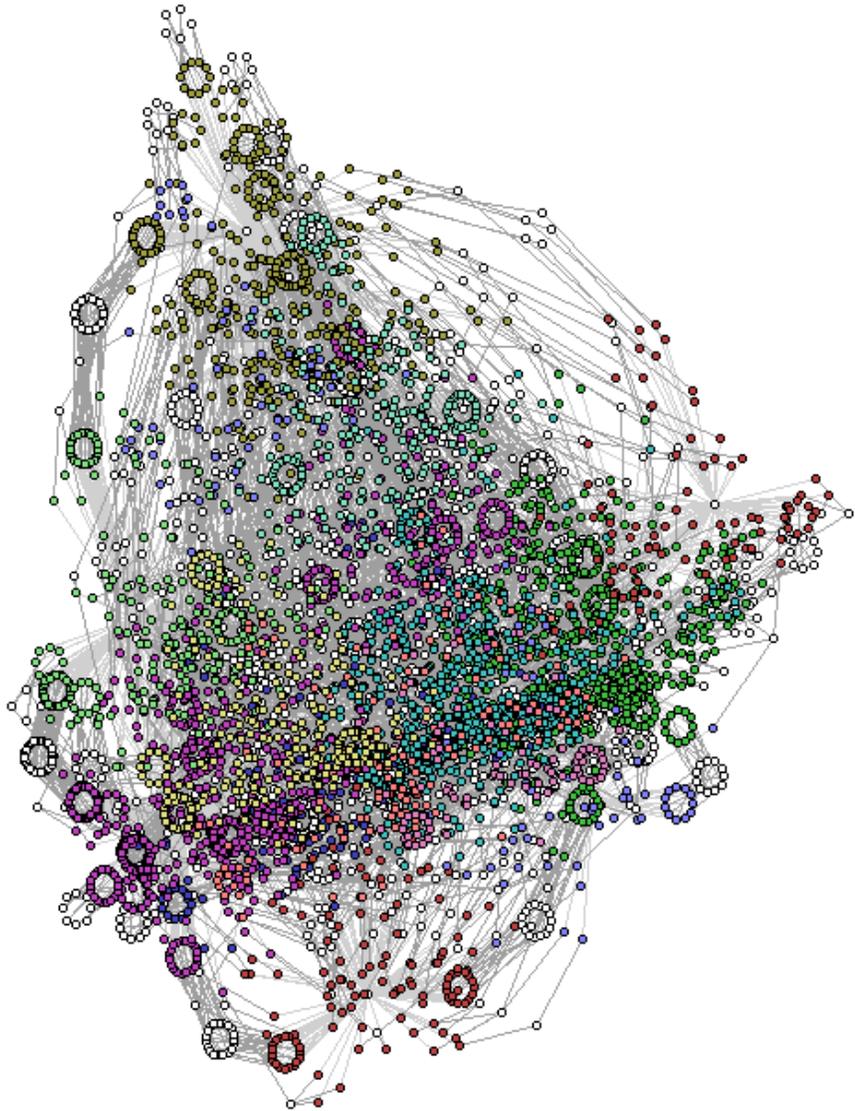
**Evec 3 Eval 0.913**

**Evec 5 Eval 0.908**

**# Nodes = 5446**

**Top 20 Cancers**

- 1001 Head/Neck**
- 1002 Stomach**
- 1003 RtColon**
- 1004 LfColon**
- 1005 NOSColon**
- 1006 Rectum**
- 1008 Liver**
- 1010 Pancreas**
- 1013 Lung**
- 1015 Leukemia**
- 1016 Skin**
- 1018 Breast**
- 1020 Cervix**
- 1021 Endometrium**
- 1022 Ovary**
- 1023 Prostate**
- 1025 Renal**
- 1026 Bladder**
- 1027 Brain**
- 1030 Lymph**



**Figure 3**